An effective interaction spontaneously arising in a renormalizable model of quantum field theory

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Theory of massless scalar field ϕ with interaction $g\phi^3$ in six-dimensional space is considered. A possibility of initial scale invariance breaking, which results in a spontaneous arising of effective interaction $G\,\phi^4$, is studied by application of Bogolubov quasi-averages approach. It is shown, that compensation equation for form-factor of this interaction in approximation up to the third order in G has a non-trivial solution. In the same approximation Bethe-Salpeter equation for a zero-mass bound state of two scalar fields ϕ is shown to have a solution. The conditions imposed on form-factor value at zero and scalar field mass m fix the unique solution, which gives relations between parameters of interaction $g\phi^3$ and parameters G and m. Arguments are laid down in favour of a stability of the non-trivial solution.

Key words: Effective interaction, quantum field theory, Bogolubov quasi-averages approach, compensation equation, non-trivial solution

1 Compensation equation in quasi-averages approach

N.N. Bogolubov quasi-averages method [1, 2] is the most consistent and effective method of studying of a spontaneous symmetry breaking problems.

An important point of the quasi-averages method is connected with a compensation equation [1, 2]. Bearing in mind applications in the present work let us briefly formulate method of construction of the compensation equations. In the line of a study of a possible spontaneous symmetry breaking in quantum field theory problems in method [2] the following

procedure is applied[1]. Let the initial Lagrangian

$$L \,=\, L_0 \,+\, L_{int}\,; \tag{1}$$

to possess some symmetry. Let us add to expression (1) some term $\epsilon\, L_{br}$, which breaks the initial symmetry. With this modification of the problem we perform evaluations of necessary quantities and we set $\epsilon \to 0$ only after these evaluations. Not always the results of such a procedure (quasi-averages) coincide with results, obtained in the framework of the initial symmetric problem (simply averages). In the line of these evaluations of quasi-averages one has to solve compensation equations. For instance, in a theory with the initial chiral symmetry fermions are to have zero masses. Let us use the following small increment which breaks the symmetry

$$\epsilon\, L_{br} \,=\, -\,\epsilon\,\bar\psi\,\psi\,. \tag{2}$$

Now let us add to the modified Lagrangian (2) a possible mass term and subtract the same. We have

$$L \,=\, L_0 \,-\, m\,\bar\psi\,\psi \,+\, L_{int} \,+\, m\,\bar\psi\,\psi \,-\, \epsilon\,\bar\psi\,\psi \tag{3}$$

Let the fist two terms to be the new free Lagrangian while the three last terms now comprise the new interaction Lagrangian. Then we have to demand the new interaction does not contribute to the mass term, that is two-field Green function obtained from the modified interaction Lagrangian be zero on the mass shell. This condition is just the compensation equation of the problem. In the case under consideration this condition leads to equation

$$-\,m \,+\, \epsilon \,+\, \Sigma(m) \,=\, 0\,; \tag{4}$$

where $\Sigma(m)$ is mass operator on the mass shell of the modified free Lagrangian. In this equation one already can set $\epsilon \to 0$. As a rule (see e.g. [3]) mass operator $\Sigma(m)$ is proportional to m and trivial solution of the compensation equation $m\,=\,0$ always exists. However nontrivial solutions $m \neq 0$ also may exist.

Thus the main principle of construction of a compensation equation consist in the procedure "add – subtract" of symmetry breaking terms, one of these terms being related to the free Lagrangian and the other one being related to the interaction Lagrangian. Then one has to compensate that term, which is to be zero in the corresponding problem. This principle will be applied in the present work.

2 Justification of the model choice

The phenomenon of spontaneous symmetry breaking is decisive for formulation of the electroweak theory. The introduction of elementary scalar fields [4] is well-known to be the essence of the standard way of the breaking. Their self-interaction leads to redefinition of

[1] At first methods [1, 2] where applied to quantum theory problems in work [3]

the vacuum in the theory and to existence of the scalar Higgs particle. However proposals based on a dynamical breaking of the initial symmetry without elementary Higgs scalars are also considered. This leads to effective four-fermion interaction of heavy quarks either to be postulated (see e.g. review [5]) or to be dynamically arisen [6, 7]. As a result the initial theory, which contains scale-invariant gauge interactions, transforms into a theory, which contains interactions with dimensional coupling constant as well, that explicitly breaks the scale invariance. In works being mentioned [6, 7] assumption was made on a possibility of existence of solutions of corresponding compensation equations. However this assumption was not duly justified. The purpose of the present work is to consider a simpler model, which would allow to have exact solutions of (approximate) compensation equations. Using these solutions one could study conditions under which the assumptions would be fulfilled. To some extent the model has to correspond to features of the electroweak theory. Namely we achieve a simplicity by considering a scalar field. In view of coupling constants to have proper dimensions we choose dimensionality of the space-time to be six. Really, in this case the coupling constant of interaction $g\,\phi^3$ is dimensionless and interaction $G\,\phi^4$ has constant of inverse mass squared dimension, that corresponds to the dimension of a constant of a four-fermion interaction in four-dimensional space-time.

So we introduce in the six-dimensional space-time a scalar field ϕ with initiative scale-invariant Lagrangian

$$L\,=\,\frac{1}{2}\,g^{\mu\nu}\,\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu}\,+\,\frac{g_0}{3!}\,\phi^3\,. \tag{5}$$

Let us choose the natural signature with one time and five space axes. The transition from this space-time to Euclidean six-dimensional space is accompanied by the following substitutions

$$p^2\,\rightarrow\,-\,p_E^2\,;\qquad d^6\,p\,\rightarrow\,\imath\,d_E^6\,p\,. \tag{6}$$

It was important for us to find a model, which corresponds to the approach under consideration. So in the present work we will not discuss physical meaning of a multi-dimensional theory and we consider the chosen variant as purely model one, as well as two-dimensional models are often considered.

Now we start with Lagrangian (5). Evident evaluations give one-loop renormalization group equation [8] for $g^2(\mu^2)$

$$\frac{d\,g^2(\mu^2)}{dL}\,=\,-\,\frac{3\,g^4}{4\,(4\,\pi)^3}\,;\qquad L\,=\,\log\frac{\mu^2}{\Lambda_3^2}\,; \tag{7}$$

Solution of equation (7) has a form

$$g^2(\mu^2)\,=\,g_0^2\,\Big(1\,+\,\frac{3\,g_0^2}{4\,(4\,\pi)^3}\,\log\frac{\mu^2}{\Lambda_3^2}\Big)^{-1}\,. \tag{8}$$

Sometimes it is convenient to use parameter $\bar{h}(\mu^2)$ defined by the following relation

$$\bar{h}(\mu^2)\,=\,\frac{3\,g^2(\mu^2)}{4\,(4\,\pi)^3}\,=\,\Big(\log\frac{\mu^2}{\Lambda_g^2}\Big)^{-1}\,; \tag{9}$$

where for transition from Λ_3^2 to Λ_g^2 we have used the standard tool analogous to that in QCD:

$$\Lambda_g^2 \,=\, \Lambda_3^2\, \exp\biggl(-\,\frac{4\,(4\pi)^3}{3\,g_0^2}\biggr)\,.$$

Thus we get convinced, that the theory (5) is an asymptotically free one and expression (9) makes sense for $\mu^2 \,\gg\, \Lambda_g^2$.

Note that in this theory there are quadratic divergences in the scalar field mass. It is the common feature of theories with elementary scalars. The problem of mass of the scalar field will be considered in details later on.

3 Compensation equation in a six-dimensional scalar model

Let us have a massless scalar field of the six-dimensional space. The initial free Lagrangian possesses scale symmetry. We shall look for a solution, which breaks this symmetry, with the aid of Bogolubov quasi-averages approach [2]. Namely according to [2] we add to the Lagrangian the following small increment

$$-\,\epsilon\,\frac{\phi^4}{4!}\,.$$

Now the scale invariance is already broken and an appearance of nonlocal terms of the form

$$G\int \bar{F}(x,x_1,x_2,x_3,x_4)\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\,dx_1\,dx_2\,dx_3\,dx_4; \qquad (10)$$

is possible. Here G is a dimensional coupling constant and $\bar{F}(x,x_1,x_2,x_3,x_4)$ is a function of four differences of coordinates $x-x_i$, which Fourier transform $F(p_1,p_2,p_3,p_4)$, where p_i are momenta of legs, represents a form-factor, defining range of interaction (10). We shall look for a solution, decreasing at momentum infinity and thus defining a region of action of the effective interaction.

Let us add to the initial free Lagrangian such a term with an interaction of the forth power and subtract the same

$$L \,=\, \frac{1}{2}\,g^{\mu\nu}\,\frac{\partial\phi}{\partial x^\mu}\,\frac{\partial\phi}{\partial x^\nu}\,-\,\frac{m^2}{2}\,\phi^2\,-\,\frac{G}{4!}\,F\cdot\phi^4\,\,-\,\frac{\epsilon}{4!}\phi^4\,+\,\frac{g_0}{3!}\,\phi^3\,+\,\frac{G}{4!}\,F\cdot\phi^4\,+\,\frac{m^2}{2}\,\phi^2\,; \qquad (11)$$

where we use abbreviated notation $-\,G\,F\cdot\phi^4$ instead of expression (10). Of course the presence of term (10) explicitly breaks the scale invariance, so we perform a procedure "add – subtract" for a mass term as well. Let us refer the forth power term with the plus sign

to the interaction Lagrangian and the same term with the minus sign we refer to the free Lagrangian.

$$L_0 \,=\, \frac{1}{2}\, g^{\mu\nu}\, \frac{\partial \phi}{\partial x^\mu}\, \frac{\partial \phi}{\partial x^\nu} \,-\, \frac{m^2}{2}\, \phi^2 \,-\, \frac{G}{4!}\, F \cdot \phi^4 \,-\, \frac{\epsilon}{4!} \phi^4\,;$$
$$L_{int} \,=\, \frac{g_0}{3!}\, \phi^3 \,+\, \frac{G}{4!}\, F \cdot \phi^4 \,+\, \frac{m^2}{2}\, \phi^2\,; \qquad (12)$$

According to the quasi-averages approach [2] the term with the plus sign has to be compensated. This means, that the new free Lagrangian leads to zero four-particle connected Green functions and as a final result contains only terms of the second power in fields. Thus performing evaluations with sign which is inherent to the term in the new free Lagrangian, we come to the compensation equation, which schematically looks in the following way: the first order term plus one-loop terms plus two-loop terms etc. Emphasize once more, that here one has to use term $+\, G\, \phi^4$ as an interaction Lagrangian. One has to equalize to zero the expansion obtained in such a way . This condition is an equation for function $F(p_1, p_2, p_3, p_4)$. We set $\epsilon \to 0$ after evaluations, in our case this means after compensation equations being obtained.

The equation explicitly differs from expansion in powers of interaction Lagrangian

$$L_{int} \,=\, \frac{G}{4!}\, F \cdot\, \phi^4\,; \qquad (13)$$

in the sign of the interaction constant. In view of this note let us emphasize, that the procedure being described can be applied only to symmetry breaking terms of even powers in fields. For terms of odd powers, e.g. for three-linear ones, a fulfillment of a compensation equation leads to vanishing of connected Green function, which is defined by an interaction Lagrangian, because the two expansions in this case differ only in overall sign.

Note, that the presence of term $-\, G\, \phi^4$ in the new free Lagrangian may lead to appearance of connected Green functions of higher powers in ϕ, that is of the sixth power, of the eighth power etc. Generally speaking, one has to construct a chain set of compensation equations for all these Green functions. We start with an equation for the fourth power Green function and the problem of higher Green functions will be discussed in what follows.

Let us construct an approximated equation for the fourth power connected Green function. First of all we choose the following kinematics: both left legs have zero momenta and the right ones have momenta p and $-p$. We restrict ourselves by terms up to two-loop ones inclusively. Namely, we have the first order term – the point; three terms of the second order – simple loops, i.e. a horizontal one and two vertical ones with permuted left legs; in the third order we have a horizontal and two vertical two-loop chains and six terms "wine glass": horizontal wine glasses having bases to the left and to the right and vertical ones with bases up and down. The number of the last terms is to be counted twice due to permutations of the left-sided momenta p and $-p$. Generally speaking, in each vertex form-factor F is present. However we can solve only a linear version of the equation, which is obtained

by keeping in the equation the first and the second order terms, the two-loop horizontal chain and the wineglass with the basing to the right. Contributions of the rest third order terms we shall consider later on. We proceed to the linear equation keeping form-factor $F(p,-p,0,0) \equiv F(p^2)$ in the first order term and in right-hand vertices of the horizontal loop of the second order, of the horizontal two-loop chain and of the wineglass in the third order. Other vertices in diagrams we consider to correspond to point-like interaction in which the form-factor is changed for its value at zero ($F(0) = 1$)

$$\frac{G}{4!}\,F(0)\,\phi^4 \;=\; \frac{G}{4!}\,\phi^4\,. \tag{14}$$

In vertical simple loops, which as well serve as a kernel of the integral equation, we substitute point-like vertices (14). Corresponding integrals diverge of course. In view of our search for decreasing solutions at momentum infinity for $F(p^2)$, we introduce some cut-off Λ, which existence is to be confirmed by results of a solution of the equation. In doing this we make the following substitution

$$\int_0^\infty dq^2 \;\to\; \Lambda^2\,.$$

For estimation of Λ order of magnitude we use the following definition

$$\Lambda^2 \;=\; \int_0^\infty F(y)\,dy\,; \tag{15}$$

where one of vertices is changed for the form-factor. For justification of the approach the problem of convergence of the integral in (15). We shall use the same cut-off Λ in logarithmically diverging integrals. A possible difference of an actual cut-off in these integrals from Λ leads to some change in constant term c¡ which enters into corresponding expressions. It will come clear, that the solution will not depend on a value of this constant. Thus the formulation of the equation in the framework of the accepted approximations does not contain arbitrary assumptions.

We consider the equation in six-dimensional Euclidean space with the aid of substitutions (6). In the course of evaluations one has to perform angle integrations in six-dimensional space of functions $((p-q)^2)^{-1}$ and $\log(p-q)^2$ with powers of (pq). We have (for the logarithmic case see [9])

$$\begin{aligned}
&\int \frac{d\Omega_6}{p^2+q^2-2\,p\,q\,\cos\theta} \;=\; \frac{4\,\pi^3}{3}\biggl(\Theta(x-y)\Bigl(\frac{3}{4x}-\frac{y}{4x^2}\Bigr)+\Theta(y-x)\Bigl(\frac{3}{4y}-\frac{x}{4y^2}\Bigr)\biggr)\,;\\
&\int d\Omega_6\,\log(p^2+q^2-2\,p\,q\,\cos\theta) \;=\; \frac{\pi^3}{12}\biggl(\Theta(x-y)\Bigl(\frac{8y}{x}-\frac{y^2}{x^2}+12\,\log\,x\Bigr)+\\
&+\Theta(y-x)\Bigl(\frac{8x}{y}-\frac{x^2}{y^2}+12\,\log\,y\Bigr)\biggr)\,; \qquad (16)\\
&\int d\Omega_6\,(p\,q)\,\log(p^2+q^2-2\,p\,q\,\cos\theta) \;=\; \frac{\pi^3}{18}\biggl(\Theta(x-y)\Bigl(\frac{3y^2}{x}-6\,y-\frac{3y^3}{5x^2}\Bigr)+
\end{aligned}$$

$$
+\,\Theta(y-x)\Big(\frac{3x^2}{y} - 6\,x - \frac{3x^3}{5y^2}\Big)\Big)\,;
$$
$$
x\,=\,p^2\,;\qquad y\,=\,q^2\,.
$$

First of all let us calculate one-loop integral keeping terms of zero and the first orders in m^2. We have for one such vertical diagram ($x = p^2$, where p is the total momentum along the loop)

$$
-\,\imath\,\frac{G^2\,\pi^3}{2\,(2\pi)^6}\left(\Lambda^2\,+\,\frac{1}{3}\,x\,\log\Big(\frac{x}{\Lambda^2}\Big)\,+2\,m^2\,\log\Big(\frac{x}{\Lambda^2}\Big)\,-\,c\,x\right);\tag{17}
$$

where Λ is the square of the cut-off being mentioned and c is a constant, depending on a behaviour of the form-factor.

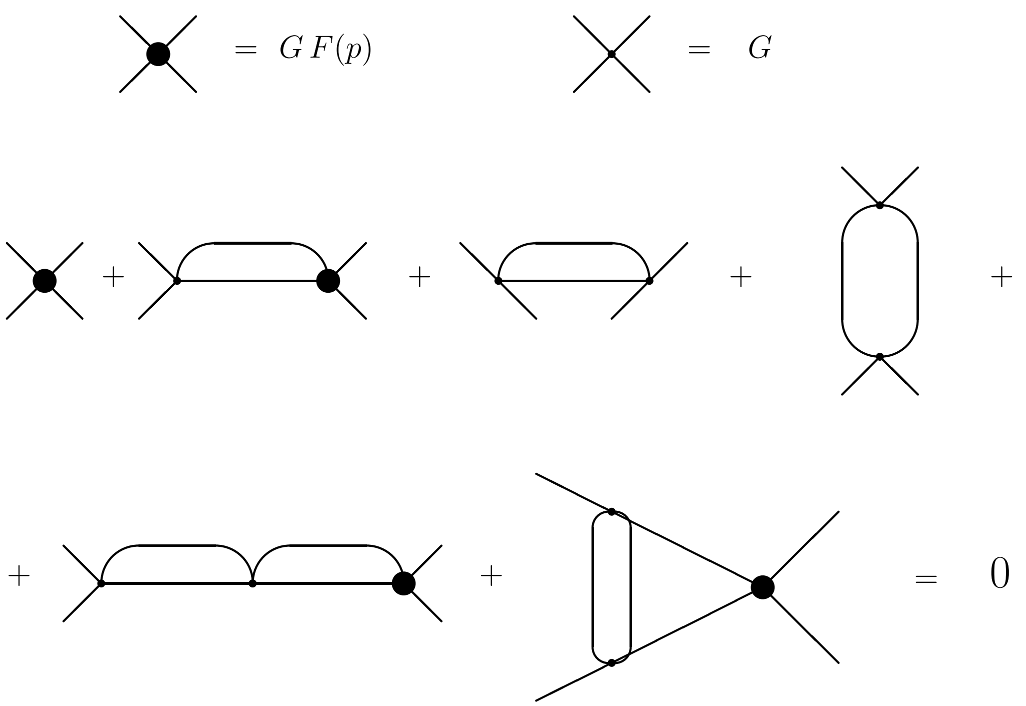

Fig. 1. The graphic representation of the linear compensation equation (18).

Let us consider the linear compensation equation, obtained in agreement with the formulated rules (see Fig. 1). The equation in this approximation has the following form

$$
\begin{aligned}
G\,F(p^2)\;=\;&\frac{G^2}{2(4\pi)^3}\left(3\,\Lambda^2\,+\,\frac{2}{3}\,p^2\,\log\Big(\frac{p^2}{\Lambda^2}\Big)\,+4\,m^2\,\log\Big(\frac{p^2}{\Lambda^2}\Big)\,-\,2\,c\,p^2\right)\;-\\
&-\,\frac{G^3}{8\,(2\pi)^9}\int\Big(\frac{1}{3}(p-q)^2\,\log\frac{(p-q)^2}{\Lambda^2}\,+2\,m^2\,\log\frac{(p-q)^2}{\Lambda^2}\,-\,c\,(p-q)^2\Big)\;\times\\
&\times\,\frac{F(q^2)}{(q^2+m^2)^2}\,d^6q\,-\,\frac{3\,G^3\,\pi^3\,\Lambda^2}{2\,(2\pi)^{12}}\int\frac{F(q^2)}{(q^2+m^2)^2}\,d^6q\,.
\end{aligned}\tag{18}
$$

Firstly let us note, that trivial solution $G = 0$ is evidently possible. In view of looking for a non-trivial solution we cancel the equation by G. Performing here angle integrations by using formulas (16) we obtain the following one-dimensional integral equation

$$
\begin{aligned}
&F(x) = \frac{G}{2(4\pi)^3}\biggl(3\Lambda^2 + \frac{2}{3}\,x\,\log\Bigl(\frac{x}{\Lambda^2}\Bigr) + 4m^2\,\log\Bigl(\frac{x}{\Lambda^2}\Bigr) - 2cx\biggr) - \\
&-\frac{3\,G^2\Lambda^2}{4(4\pi)^6}\int_0^\infty \frac{y^2\,F(y)}{(y+m^2)^2}\,dy \;-\; \frac{G^2}{18(4\pi)^6}\biggl(-\frac{1}{20\,x^2}\int_0^x \frac{y^5\,F(y)}{(y+m^2)^2}\,dy\;+ \\
&+\frac{3}{4x}\int_0^x \frac{y^4\,F(y)}{(y+m^2)^2}\,dy \;+\; 3\,\log\,x\int_0^x \frac{y^3\,F(y)}{(y+m^2)^2}\,dy \;+\; 3\,x\,\log\,x\int_0^x \frac{y^2\,F(y)}{(y+m^2)^2}\,dy\;+ \\
&+3\int_x^\infty \frac{y^3\,\log\,y\,F(y)}{(y+m^2)^2}\,dy \;+\; x\int_x^\infty \frac{(4+3\log\,y)\,y^2\,F(y)}{(y+m^2)^2}\,dy\;+ \\
&+4\int_0^x \frac{y^3\,F(y)}{(y+m^2)^2}\,dy \;+\; \frac{3\,x^2}{4}\int_x^\infty \frac{y\,F(y)}{(y+m^2)^2}\,dy \;-\; \frac{x^3}{20}\int_x^\infty \frac{F(y)}{(y+m^2)^2}\,dy\biggr)\;+ \\
&-\frac{G^2\,m^2}{12(4\pi)^6}\biggl(-\frac{1}{x^2}\int_0^x \frac{y^4\,F(y)}{(y+m^2)^2}\,dy + \frac{8}{x}\int_0^x \frac{y^3\,F(y)}{(y+m^2)^2}\,dy + 12\,\log x\int_0^x \frac{y^2\,F(y)}{(y+m^2)^2}\,dy\,+ \\
&+12\int_x^\infty \frac{y^2\,\log\,y\,F(y)}{(y+m^2)^2}\,dy \;+\; 8\,x\int_x^\infty \frac{y\,F(y)}{(y+m^2)^2}\,dy \;-\; x^2\int_x^\infty \frac{F(y)}{(y+m^2)^2}\,dy\biggr)\;+ \\
&+\frac{G^2}{6\,(4\pi)^6}\Bigl(\log\,\Lambda^2 \;+\; 3\,c\Bigr)\biggl(\int_0^\infty \frac{y^3\,F(y)}{(y+m^2)^2}\,dy \;+\; x\int_0^\infty \frac{y^2\,F(y)}{(y+m^2)^2}\,dy\biggr)\;+ \\
&+\frac{G^2\,m^2}{(4\pi)^6}\;\log\,\Lambda^2\int_0^\infty \frac{y^2\,F(y)}{(y+m^2)^2}\,dy\,.
\end{aligned}
\tag{19}
$$

A method of solution of equations of (19) type is developed in work [11]. Equation (19) is reduced to a differential one by sequential differentiations. Evident evaluation gives

$$
\begin{aligned}
&\frac{d^4}{dx^4}\biggl(x^2\frac{d^4}{dx^4}\Bigl(x^2F(x)\Bigr)\biggr) = -\,\beta\biggl(\frac{F(x)}{(x+m^2)^2} + 2m^2\Bigl(x\,\frac{d^2}{dx^2}\frac{F(x)}{(x+m^2)^2} + 3\frac{d}{dx}\frac{F(x)}{(x+m^2)^2}\Bigr)\biggr);\\
&\beta \;=\; \frac{2\,G^2}{(4\,\pi)^6}\,.
\end{aligned}
\tag{20}
$$

One easily see, that Eq. (20) can be rewritten in the form

$$
\begin{aligned}
&\biggl(\Bigl(x\,\frac{d}{dx}\,+\,2\Bigr)\Bigl(x\,\frac{d}{dx}\,+\,1\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\;\times \\
&\times\Bigl(x\,\frac{d}{dx}-2\Bigr)\Bigl(x\frac{d}{dx}-3\Bigr)\;+\;\beta\,x^2\biggr)\,F(x)\;=\;2\,\beta\,m^2x\Bigl(F(x)+x\frac{d\,F}{dx}-x^2\frac{d^2\,F}{dx^2}\Bigr);
\end{aligned}
\tag{21}
$$

where two terms of expansion in m^2 are kept. From this form of the equation we immediately conclude, that for $x\,\to\,0$ there are eight independent asymptotes, which coefficients we

denote as follows

$$\frac{a_{-2}}{x^2};\quad \frac{a_{-1}}{x};\quad a_0\,;\quad a_{0l}\,\log\,x;\quad a_1\,x;$$
$$a_{1l}\,x\,\log\,x;\quad a_2\,x^2;\quad a_3\,x^3\,. \tag{22}$$

Eight independent asymptotes at infinity are the following

$$F_k(x)\,\simeq\,x^{-3/8}\,\exp\biggl(4\,(\beta\,x^2)^{1/8}\exp\Bigl(\frac{\imath\,\pi\,(2k-1)}{8}\Bigr)\biggr)\,;\qquad k\,=\,1,\,2,\,...,\,8. \tag{23}$$

Four of these asymptotes at infinity decrease exponentially ($k\,=\,3,\,4,\,5,\,6$), and the rest four ones do increase.

Equation (21) is equivalent to the initial integral equation under definite boundary conditions being fulfilled. First of all we can use only solutions, decreasing at infinity. To obtain conditions at zero we have to substitute expression

$$F(x)\,=\,-\,\frac{x^2}{\beta}\,\frac{d^4}{dx^4}\biggl(\,x^2\,\frac{d^4}{dx^4}\,\Bigl(\,x^2\,F(x)\,\Bigr)\biggr)\,;$$

in integrals of equation (19) and perform sequential integrations by parts. The results are presented in the Appendix.

Substituting expressions (57) into equation (19), we have

$$F(x)\,=\,F(x)\,-\,\frac{a_{-2}}{x^2}\,-\,\frac{a_{-1}}{x}\,-\,a_{0l}\,\log\,x\,-\,a_{1l}x\,\log\,x\,+$$
$$+\,\frac{G\,\pi^3}{2(2\pi)^6}\,\biggl(3\,\Lambda^2\Bigl(1-\frac{G\,I}{2(4\,\pi)^3}\Bigr)\,+\,\frac{2\,x}{3}\,\log\Bigl(\frac{x}{\Lambda^2}\Bigr)\,-\,2\,c\,x\biggr)\,+\,x\,\Bigl(\log\,\Lambda^2\,+\,3\,c\Bigr)\,a_{1l}\,; \tag{24}$$
$$I\,=\,\int_0^\infty\frac{y^2\,F(y)}{(y+m^2)^2}\,dy\,.$$

From here we obtain the following condition (independently on values of Λ^2 and c)

$$a_{-2}\,=\,0\,,\qquad a_{-1}\,=\,0\,,\quad a_{0l}\,=\,\frac{2\,G\,m^2}{(4\pi)^3}\,,$$
$$a_{1l}\,=\,\frac{G\pi^3}{3\,(2\pi)^6}\,=\,\frac{\sqrt{2\,\beta}}{6}\,; \tag{25}$$
$$I\,=\,\frac{2(4\pi)^3}{G}\,=\,\frac{2\,\sqrt{2}}{\sqrt{\beta}}\,. \tag{26}$$

The first four conditions (25) are boundary conditions for Eq. (21). A combination of four solutions decreasing at infinity with account of these boundary conditions gives the unique

solution. It can be expressed in terms of well-known special functions for case $m^2 = 0$. Indeed, let us make the following substitution in Eq. (21)

$$z = \frac{\beta x^2}{2^8}; \tag{27}$$

which reduces the equation to the canonical form of Meijer equation [12] of the eighth order

$$\Bigl(\Bigl(z\frac{d}{dz}+1\Bigr)\Bigl(z\frac{d}{dz}+\frac{1}{2}\Bigr)\Bigl(z\frac{d}{dz}\Bigr)\Bigl(z\frac{d}{dz}\Bigr)\Bigl(z\frac{d}{dz}-\frac{1}{2}\Bigr)\Bigl(z\frac{d}{dz}-\frac{1}{2}\Bigr)\times$$
$$\times\Bigl(z\frac{d}{dz}-1\Bigr)\Bigl(z\frac{d}{dz}-\frac{3}{2}\Bigr)+z\Bigr)F(z) = 0. \tag{28}$$

Conditions (25) fix the solution. Firstly, four solutions, decreasing at infinity, always could be combined to set to zero three singular asymptotes at zero, i.e. to fulfill conditions $a_{-2} = a_{-1} = a_{0l} = 0$. Such property has the following Meijer function (see [12])

$$C \cdot G^{50}_{08}(\, z \,|\, 3/2,\, 1,\, 1/2,\, 1/2,\, 0,\, 0,\, -1/2,\, -1).$$

The constant is defined by the coefficient afore $\sqrt{z}\log z$. For small z this Meijer function is such [12]

$$G^{50}_{08}(z \,|\, 3/2,\, 1,\, 1/2,\, 1/2,\, 0,\, 0,\, -1/2,\, -1) = \pi + \frac{16}{3}\sqrt{z}\,\log z + \dots. \tag{29}$$

Comparing the coefficient afore $\sqrt{z}\,\log z$ with (25), we obtain

$$C = \frac{\sqrt{2}}{4}.$$

Performing integration (see [13]), we have in accordance with definition of I (24)

$$I = \int_0^\infty F(y)\,dy = \frac{\sqrt{2}}{4}\int_0^\infty G^{50}_{08}(\beta y^2/2^8 \,|\, 3/2, 1, 1/2, 1/2, 0, 0, -1/2, -1)\,dy = \frac{2\sqrt{2}}{\sqrt{\beta}}; \tag{30}$$

that perfectly agrees with condition (25).

Thus, solution

$$F(x) = \frac{\sqrt{2}}{4}\,G^{50}_{08}(\beta x^2/2^8 \,|\, 3/2,\, 1,\, 1/2,\, 1/2,\, 0,\, 0,\, -1/2,\, -1)\,; \tag{31}$$

fulfills all conditions (25), and consequently the initial equation (19), which is an approximate compensation equation. This solution is a nontrivial solution, which contains dimensional parameter G, and hence it leads to the initial scale symmetry breaking. Of course as we have noted before trivial solution $F(x) = 0$ is also possible. Note, that the boundary conditions

are not dependent on value of the form-factor at zero. Equality $F(0) = 1$ will serve as an additional condition in what follows.

Let us take into account terms proportional to m^2. We shall look for a correction to the solution of Eq. (21) in the following form

$$F(x) \,=\, F_0(x) \,+\, \Delta\, F(x)\,. \tag{32}$$

Substituting (32) into equation (21) we have the following equation in the first order in m^2

$$\begin{aligned}&\biggl(\Bigl(x\,\frac{d}{dx}\,+\,2\Bigr)\Bigl(x\,\frac{d}{dx}\,+\,1\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,2\Bigr)\,\times\\&\times\Bigl(x\,\frac{d}{dx}\,-\,3\Bigr)\,+\,\beta\,x^2\biggr)\,\Delta\,F(x)\,=\,2\,\beta\,m^2\,x\,\Bigl(F_0(x)\,+\,x\,\frac{d\,F_0}{dx}\,-\,x^2\,\frac{d^2\,F_0}{dx^2}\Bigr).\end{aligned} \tag{33}$$

From equation (33) we can exactly define several terms of expansion of $\Delta\, F(x)$ for small x. Indeed let us consider the following expression

$$\begin{aligned}\bar{\Delta}\,F(x)\,&=\,\frac{2\,m^2}{x}\,\Bigl(F_0(x)\,+\,x\,\frac{d\,F_0}{dx}\,-\,x^2\,\frac{d^2\,F_0}{dx^2}\Bigr)\,=\\&=\,2\,m^2\biggl(\frac{\pi}{2\sqrt{2}\,x}+\frac{2\,G}{3\,(4\pi)^3}\Bigl(\log\,(\sqrt{\beta}\,x)\,+4\,\gamma-\frac{23}{6}\Bigr)\,+\,\frac{\pi\sqrt{2}\,G^2}{96\,(4\pi)^6}\,x\,\log\,x+O(x)\biggr)\,;\end{aligned} \tag{34}$$

where $\gamma \,=\, 0.577215665...$ is the Euler constant. Substituting expression (34) into equation (33), we get convinced, that it fulfills the equation up to terms of x^3 order, because the differential operator in the left-hand side nullifies the terms presented in (34) and subsequent terms up to the indicated order. We are interested just in the presented terms (34) because they refer to the boundary conditions. Indeed expression (34) contains terms $z^{-1/2}$, $\log z$, $z^{1/2}\,\log\,z$, which violate their boundary conditions. Hence we are to add to expression (34) a combination of solutions of the homogeneous equation to force the boundary conditions to be fulfilled. Finally we obtain

$$\begin{aligned}&\Delta\,F(x)\,=\,\bar{\Delta}\,F(x)\,-\,\frac{\pi^2\,Y}{8}\,G^{50}_{08}(\beta\,x^2/2^8\,|\,3/2,\,1,\,1/2,\,1/2,\,-1/2,\,0,\,0,\,-1\,)\,-\\&-\,\frac{2\,Y}{3}\,G^{50}_{08}(\beta\,x^2/2^8\,|\,3/2,\,1,\,1/2,\,0,\,0,\,1/2,\,-1/2,\,-1\,)\,-\\&-\,\pi\,Y\,\Bigl(\gamma\,+\,\log\,2\,-\,\frac{43}{48}\Bigr)\,G^{50}_{08}(\beta\,x^2/2^8\,|\,3/2,\,1,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1\,)\,;\\&\quad Y\,=\,\frac{G\,m^2}{2\,(4\pi)^3}\,.\end{aligned} \tag{35}$$

From this expression we extract the exact value for $F(0)$. While doing this one has to bear in mind, that the presence of a term being proportional to $\log\,x$ at $x \to 0$ is a consequence

of an expansion in m^2 at $x \gg m^2$. Looking back at the corresponding evaluations we see, that for $x \to 0$ one has to change $\log x$ for $\log 4\, m^2$. Now we have

$$F(0) \,=\, \frac{\pi\,\sqrt{2}}{4} + Y\left(4\log Y + (16-\pi^2)\gamma + (14-\pi^2)\log 2 - \frac{122}{9} + \pi^2\frac{42}{48}\right). \tag{36}$$

For $Y \,=\, 0$ we obtain $F(0) \,=\, 1.11072$. Condition $F(0) \,=\, 1$ defines the value of Y, which is connected with mass (see (35))

$$Y \,=\, 0.005789\,. \tag{37}$$

Thus the solution, which is found here, satisfies all the necessary conditions provided (37) is valid. Emphasize, that (37) defines the mass of the scalar field. Note, that the small value of (37) thoroughly justifies the account of only the first term of the expansion in m^2. We reject the second solution of condition $F(0) = 1$, which is of order of unity, due to to its inconsistence with the expansion of the solution in m^2.

We have mentioned already, that generally speaking one has to consider a total chain of compensation equations including connected Green functions with six, eight, etc. legs. Note, that corresponding equations will contain inhomogeneous parts, expressed in terms of Green functions of lower order, and homogeneous parts, being proportional to the corresponding form-factor, e.g. F_6 with six legs. Assuming our result the connected four-leg Green function be zero, we come to the conclusion, that inhomogeneous part of equation for F_6 is zero, so trivial solution $F_6 = 0$ inevitably exists. The analogous considerations lead to conclusions on possibility of existence of trivial solutions of all higher Green functions. One may, of course, study possibilities of existence of nontrivial solutions as well. However, the purpose of the present work is to show that even though one nontrivial solution does exist, so we rely on following variant: nontrivial solution for four-leg connected Green function and trivial solutions for all higher connected Green functions. The consideration of compensation equation for Green function with two legs, which defines mass of the scalar field will be performed particularly later on.

The next step of study should include non-linear equation with account of all possible diagrams. However this problem evidently do not admit analytic solution. Approximate estimate of non-linear corrections to the form-factor's value at zero will be obtained in what follows. Maybe future studies will be connected with numerical methods. We are convinced, that the experience achieved in finding of the non-trivial solution will help in formulation and realization of numerical methods. Presumably result (37), which means the existence of a solution only for definite relation between dimensional coupling constant and mass of scalar field, will be important.

4 Bethe-Salpeter equation and zero excitation

It is well-known, that a symmetry breaking is to be accompanied by an appearance of an excitation with zero mass [1, 2, 14]. Let us consider this problem in the same approximation.

While constructing an equation for a bound state one has to keep in mind, that here genuine interaction (13) acts, that one, which is referred to the interaction Lagrangian and remains, of course, not compensated. Bethe-Salpeter equation for a massless bound state of two scalar fields in this case has the form

$$\Psi(x) \,=\, \frac{G\,\pi^3\,\Lambda'}{2(2\pi)^6} \,-\, \frac{G^2\,\pi^6\,\Lambda\,\Lambda'}{2\,(2\pi)^{12}} \,+\, \frac{G^2\,\pi^6}{18\,(2\pi)^{12}}\biggl(-\,\frac{1}{20\,x^2}\int_0^x y^3\,\Psi(y)\,dy\,+$$
$$+\,\frac{3}{4x}\int_0^x y^2\,\Psi(y)\,dy\,+\,3\,\log\,x\int_0^x y\,\Psi(y)\,dy\,+$$
$$+\,3\,x\,\log\,x\int_0^x \Psi(y)\,dy\,+\,4\int_0^x y\,\Psi(y)\,dy\,+\,3\int_x^\infty y\,\log\,y\,\Psi(y)\,dy\,+$$
$$+\,x\int_x^\infty (4+3\log\,y)\,\Psi(y)\,dy\,+\,\frac{3\,x^2}{4}\int_x^\infty \frac{\Psi(y)}{y}\,dy\,-\,\frac{x^3}{20}\int_x^\infty \frac{\Psi(y)}{y^2}\,dy\biggr)\,; \qquad (38)$$
$$\Lambda' \,=\, \int_0^\infty \Psi(y)\,dy\,.$$

Comparing this equation (38) with compensation equation (19), we see the main difference in the sign afore the kernel of the integral equation. Remind once more, that the compensation equation is the condition of vanishing of the total expansion in G in the modified free Lagrangian in expression (11) and therefore terms of the first and of the third orders are situated in the same part of equation, e.g. in the left-handed one, whereas in the Bethe-Salpeter equation the corresponding terms are situated in different parts of equation.

The sign afore the kernel is very important. This means, that in a differential equation sign afore β changes as well

$$\biggl(\Bigl(x\,\frac{d}{dx}\,+\,2\Bigr)\Bigl(x\,\frac{d}{dx}\,+\,1\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,1\Bigr)\,\times$$
$$\times\,\Bigl(x\,\frac{d}{dx}\,-\,2\Bigr)\Bigl(x\,\frac{d}{dx}\,-\,3\Bigr)\,-\,\beta\,x^2\biggr)\,\Psi(x)\,=\,0\,. \qquad (39)$$

One easily see, that due to absence of term being proportional to $x\,\log\,x$ in the inhomogeneous part boundary conditions are the following

$$a_{-2}\,=\,a_{-1}\,=\,a_{0l}\,=\,a_{1l}\,=\,0\,. \qquad (40)$$

The change of sign afore β leads to changing of asymptotes at infinity

$$\Psi_k(x)\,\simeq\,x^{-3/8}\,\exp\biggl(4\,(\beta\,x^2)^{1/8}\exp\Bigl(\frac{\imath\,\pi\,k}{4}\Bigr)\biggr)\,;\qquad k\,=\,1,\,2,\,...,\,8. \qquad (41)$$

Now we have three decreasing asymptotes ($k\,=\,3,\,4,\,5$), two oscillating ones with power decreasing ($k=2,\,6$), and the remaining three are increasing. Using the first five solutions,

which allow a definition of integrals at infinity, we fulfill four boundary conditions at zero (40). AS a result we obtain the following solution of equation (38)

$$\Psi(x) \;=\; A\, G^{40}_{08}(\beta\, x^2/2^8\,|\,3/2,\,1,\,1/2,\,0,\,1/2,\,0,\,-1/2,\,-1\,)\,; \tag{42}$$

where constant A is defined by normalization condition of a Bethe-Salpeter wave function. Direct calculation [13] leads to result $\Lambda' \;=\; 0$, so the inhomogeneous part of equation (38) vanishes. Thus we have shown, that the equation for a bound state with zero mass has a solution.

The solution being obtained proves the existence of zero mass excitation [1, 2, 14] in the model. Of course definition of a Bethe-Salpeter equation itself is possible only provided a non-trivial solution of a compensation equation to exist and thus interaction (13) to act. The obligatory correspondence between a non-trivial solution of a compensation equation and an existence of a zero excitation thoroughly corresponds to Bogolubov quasi-averages approach [2].

It is interesting to note, that with taking into account of three-fold interaction $g\,\phi^3$ in the kernel of equation (38) the mass of the bound state becomes non-zero. One easily understands this, because interaction (38) itself leads to dimensional parameter Λ_3 being present and thus the scale invariance being already broken.

5 Compensation equation for scalar field mass

Let us look at interaction Lagrangian (12). The mass term there is quite improper. To solve the problem one has to formulate a compensation equation for Green function with two scalar legs. Let us consider this equation taking into account solution (32) and three-fold interaction. The compensation equation means nullification of total contribution of interaction (12) to the mass. In the first approximation the contribution of the four-fold interaction is described by the first order diagram "bubble" and that of the three-fold one is represented by simple one-loop diagram (see Fig. 2).

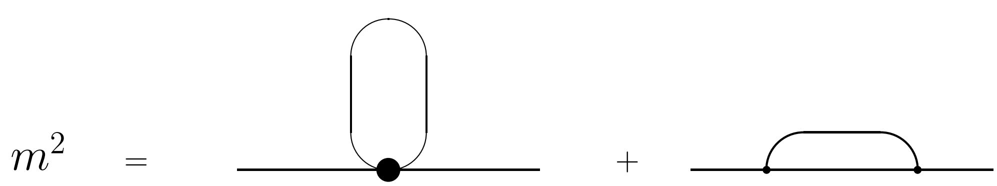

Fig. 2. Compensation equation for mass of the scalar field.

Putting momenta of the external legs to be zero, we have for "bubble" diagram just solution (32) in the vertex. As a result we obtain the following compensation equation for scalar mass

$$m^2 = -\frac{G}{(2\,\pi)^6}\int\frac{F(q^2)\,d^6q}{q^2+m^2} - \frac{g^2}{(2\pi)^6}\int\frac{d^6q}{(q^2+m^2)^2} =$$
$$= -\frac{G}{2\,(4\,\pi)^3}\int_0^\infty y\,dy\,(F_0(y) + \Delta F(y)) + \frac{Gm^2}{2\,(4\,\pi)^3}\int_0^\infty dy\,F_0(y) -$$
$$-\frac{g^2}{2\,(4\pi)^3}\int_0^\infty\frac{y^2\,dy}{(y+m^2)^2}\,; \tag{43}$$

Here in "bubble" diagram we perform an expansion in m^2 and take into account the zeroth and the first orders of the expansion. By direct evaluation with the aid of expressions (31, 34, 35) we obtain that the zeroth order terms is zero and the first order term is equal to $3\,m^2$. The loop, which is described by the last term in (43), quadratically diverges. Note, that in the initial theory (5) we introduce some cut-off Λ_3, which corresponds to a physical limitation of a region of applicability of the theory. As a result we have the following compensation equation for the mass provided $m \ll \Lambda_3$

$$m^2 = 3\,m^2 - \frac{g^2}{2\,(4\pi)^3}\,\Lambda_3^2. \tag{44}$$

Emphasize, that for the trivial solution $G = 0$ the first term in the right-hand side of equation (44) is absent and we have a negative mass squared, i.e. a tachyon solution. For the non- trivial solution we have

$$m^2 = \frac{g^2}{4\,(4\,\pi)^3}\,\Lambda_3^2\,. \tag{45}$$

It is well-known, that a scalar tachyon leads to instability for small fields. Therefore the restoration of the normal sign of the mass squared, which is achieved provided the non-trivial solution is valid, corresponds to a transition to a more stable state.

So the value of the scalar mass is defined in terms of initial parameters of the theory g and Λ_3. The value of parameter Y (37) gives the relation of the mass and of the coupling constant G of the four-fold interaction. Thus all the parameters entering into the non-trivial solution are defined in terms of the initial ones.

Note that the initial cut-off Λ_3 corresponds to some boundary energy, which provides real physical cut-off of the corresponding integrals. In the physical four-dimensional space-time it may be for example the Planck energy $1.22\cdot 10^{19}\,GeV$. Note also that in realistic models without elementary scalars (see e.g. [6], [7]) quadratic divergences in mass of the elementary fields are absent. One should expect the expressions similar to (44) also would lead to relations, which connect the theory parameters with a boundary energy (e.g. the Planck one), which enters into logarithmically divergent terms.

The final result for effective Lagrangian of the theory after the symmetry breaking occurs is the following

$$L \;=\; \frac{1}{2}\,\frac{\partial\phi}{\partial x^\mu}\,\frac{\partial\phi}{\partial x^\mu}\;-\;\frac{G}{4!}\,\bar F(x_1,x_2,x_3,x_4)\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\,; \tag{46}$$

where form-factor F is the solution of the compensation equation.

6 Estimate of non-linearity influence

Till now our results were obtained in the framework of the linear approximation. The decrease of the form-factor at infinity indicates an applicability region of the approximation. It evidently is incorrect for large momenta variables because the effective coupling constant becomes too small in comparison to constant G, which was used to define the kernel of the integral equation. We can roughly take into account an influence of a non-linearity, using the following procedure. Let equation (21) be valid for small x (we put $m^2 = 0$).

$$\frac{d^4}{dx^4}\biggl(x^2\,\frac{d^4}{dx^4}\Bigl(x^2\,F(x)\Bigr)\biggr)\;=\;-\,\beta\,\frac{F(x)}{x^2}\,;\quad \beta\;=\;\frac{2\,G^2}{(4\,\pi)^6}\,. \tag{47}$$

We use this equation with the corresponding boundary conditions (25) for $x \le x_0$, whereas for $x \ge x_0$ one has to take into account a non-linearity. Let us draw attention to the fact, that β is proportional to G^2 i.e. it contains the form-factor squared. Therefore for $x \ge x_0$ instead of (47) we use the following equation

$$\frac{d^4}{dx^4}\biggl(x^2\,\frac{d^4}{dx^4}\Bigl(x^2\,F(x)\Bigr)\biggr)\;=\;-\,\beta\,\frac{F^3(x)}{x^2}\,. \tag{48}$$

In this approximation we have correct behaviour of right-hand sides at small (47) and at very large (48) values of x. In the intermediate region there is a tear in the rhs. at $x = x_0$. This means that the eighth derivative tears at this point. As we shall see soon the form-factor and its derivatives up to the fifth order have to be continuous.

Let us introduce variable $y = \sqrt{\beta}\,x$. One easily sees that for $y \to \infty$ equation (48) defines the following decreasing asymptotics

$$F(y)\;\simeq\;\frac{b}{y^2}\;-\;\frac{6\,b^3}{5!\,7!\,y^4}\;+\;\frac{12\,b^5}{7!\,7!\,8!\,y^6}\;+\;...\,; \tag{49}$$

where b is a constant. At the same time equation (47) with account of boundary conditions has the following solution in region $(0,\,y_0)$

$$F(y)\;=\;\frac{\sqrt{2}}{4}\,G^{50}_{08}(y^2/256\,|\,3/2,\,1,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1)\;+$$

$$
\begin{aligned}
&+ C_1\, G^{30}_{08}(y^2/256\,|\,3/2,\,1,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1)\;+\\
&+ C_2\, G^{30}_{08}(y^2/256\,|\,3/2,\,1,\,0,\,1/2,\,1/2,\,0,\,-1/2,\,-1)\;+\\
&+ C_3\, G^{10}_{08}(y^2/256\,|\,3/2,\,1,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1)\;+\\
&+ C_4\, G^{10}_{08}(y^2/256\,|\,1,\,3/2,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1)\,;
\end{aligned}
\tag{50}
$$

where C_i are constants. The appearance of the additional terms with these coefficients multiplied by Meijer functions increasing at infinity is due to the fact, that now the decrease at infinity is provided by asymptotics (49) and thus in region $(0,\, y_0)$ we have to use all solutions of equation (47), which fulfill the boundary conditions at zero. The first line here is solution (31), which was obtained earlier. Let us begin a sequential account of the new terms starting from the zero approximation, in which in region $(0,\, y_0)$ we have this old solution, i.e. all $C_i\,=\,0$. This solution is matched to solution (49) in point y_0. It will come clear, that in expression (49) an account of the first term is sufficient. Then from continuity of the function and of its first derivative we obtain the following set of equations

$$
\begin{aligned}
&\frac{\sqrt{2}}{4}\, G^{50}_{08}(y_0^2/256\,|\,3/2,\,1,\,1/2,\,1/2,\,0,\,0,\,-1/2,\,-1)\;-\;\frac{b}{y_0^2}\;=\;0\,;\\
&\frac{\sqrt{2}}{4}\, G^{50}_{08}(y_0^2/256\,|\,3/2,\,1,\,1,\,1/2,\,1/2,\,0,\,-1/2,\,-1)\;-\;\frac{b}{y_0^2}\;=\;0\,;
\end{aligned}
\tag{51}
$$

Solution of the set:

$$
y_0\;=\;8.4980\,;\qquad b\;=\;7.5055\,. \tag{52}
$$

The second term in asymptotics (49) at y_0 comprises $7.7\cdot 10^{-6}$ times the first one, that justifies the account of the first term only. The value of the form-factor at zero does not change $F(0)\;=\;1.1107$.

Now let us take into account two additional terms in (50) with coefficients C_1 and C_2, which for small y give larger contribution than the remaining two terms. In this case we have to match values of the function and of its derivatives up to the third order. One obtains the set of four equations with aid of rules of differentiation of Meijer functions [12]. Its solution reads

$$
y_0\;=\;17.635\,;\quad b\;=\;9.410\,;\quad C_1\;=\;0.0166\,;\quad C_2\;=\;-0.0538\,. \tag{53}
$$

The value of the form-factor at zero becomes the following

$$
F(0)\;=\;\frac{\pi\,\sqrt{2}}{4}\;+\;\frac{C_2}{\pi}\;=\;1.0936\,. \tag{54}
$$

Now let us take into account terms with coefficients C_3, C_4. We consider them and deviations from solution (53) as well to be small. Then matching the function and its derivatives up to the fifth order, we obtain a set of six linear equations leading to the following solution

$$
\begin{aligned}
&\Delta y_0\;=\;1.457,\qquad \Delta b\;=\;1.032,\qquad \Delta C_1\;=\;-\,0.0094,\\
&\Delta C_2\;=\;0.0223,\qquad C_3\;=\;-\,0.0249,\qquad C_4\;=\;0.0136\,.
\end{aligned}
\tag{55}
$$

Substituting the last result into (54), we have

$$F(0) \,=\, 1.1007\,. \tag{56}$$

The sequence of numbers 1.1107, 1.0936, 1.1007 for value $F(0)$ demonstrates stability of the result in respect to contribution of non-linear corrections

7 Conclusion

Grounding on the results being obtained we express a hypothesis, that in the model under consideration a nontrivial solution does exist, which breaks the initial scale invariance and leads to a spontaneous appearance of effective interaction in Lagrangian (46), acting in a restricted region of the momenta space in accordance with the value of parameter G. Effective form-factor $F(p)$ decreases exponentially with oscillations for $p^2 \,\to\, \pm\,\infty$, i.e. both for space-like and time-like momenta. We confirm the existence of a zero mass excitation, which has to be present for an occurrence of spontaneous symmetry breaking.

We start with the asymptotically free theory of a scalar field (in a six-dimensional space), and we obtain as a result the definite theory with interaction breaking scale symmetry. New dimensional parameters $G^{-1/2}$ and m are proportional to parameter Λ_3, which defines the initial asymptotically free interaction. Let us emphasize once more, that the interaction being obtained is an effective one, that first of all is reflected in a presence of form-factor $F(p)$, which is just the solution of compensation equation of N.N. Bogolubov method. At momentum infinity the theory becomes asymptotically free again.

It is quite important, that the problem under consideration has a consistent solution only provided triple interaction $g\,\phi^3$ is acting. Really, albeit compensation equation (18) contains no contribution of this interaction, the non-zero scalar field mass appears only for $g\,\neq\,0$. If it is not the case the value of form-factor at zero $F(0)$ is not unity. In general one can not exclude a possibility of condition $F(0) = 1$ being fulfilled for $m = 0$. However the experience obtained in considering the present problem shows that this condition could be fulfilled only provided the model has very peculiar properties. As a matter of fact the problem under consideration is defined not by compensation equation (18) only, but by set of equations (18, 43), which explicitly contains a contribution of triple interaction $g\,\phi^3$.

It should be noted, that a possibility of a nontrivial solution strongly depends on the choice of the theory. This may be demonstrated by comparison of different signatures of the six-dimensional space-time. Namely if one instead of signature 1 + 5 will choose signature 3 + 3, then in definition (6) of a transition to Euclidean coordinates the sign afore $\imath\,d^6p$ changes. As a result all signs change for one-loop integrals. For four-fold interaction we restore all previous results by simple substitution $G \to -\,G$. However the one-loop integral with two three-fold vertices inevitably changes sign and relation (44) leads to tachyon mass. So we come to the conclusion, that for signature 3 + 3 only the trivial solution $G \,=\, 0$ is stable.

Of course, we base our conclusions only on exact solutions of approximate equations. However it is possible, that qualitative properties of solutions, which manifest themselves in the model problem, will be quite useful in study of problems of spontaneous symmetry breaking in more realistic cases, when there is no hope for analytic solution of corresponding equations and what is possible to apply are just numerical methods. Attractive qualitative results are the existence of relations between parameters of the problem and the natural appearance of small parameter Y (37). The essential result is connected also with the conclusion on the stability of the non-trivial solution. The estimate of non-linearity contribution, which does not lead to decisive change of properties of the solution, provides additional argument on behalf of the present approach.

The resulting theory is non-local and the question might arise, whether the general principles of unitarity and causality are here valid. The initial theory (5) quite corresponds to these principles. One should expect, that its solutions, non-trivial ones as well, have also to fulfill these conditions. Therefore one can consider the present example as a step in direction of formulating of a consistent non-local theory. Basing on results of the present work we may assume, that such theory can be consistent not for an arbitrary form-factor but for the one, which follows from a non-trivial solution of an initially local theory.

Without any doubt a possibility of spontaneous appearance of an effective interaction, containing a dimensional parameter, is of great interest for studies of problems beyond the standard theory. In particular, the phenomenon of a spontaneous appearance of an effective interaction, provided it to occur in a genuine physical theory, e.g. in the electroweak theory, might essentially promote our understanding of bases of the theory. Some considerations in connection with this aspects are described in work [6, 7]. A subsequent results in this direction and their connection with possible deviations from predictions of the standard theory will be presented elsewhere.

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A Appendix

Here formulas of integration by parts of expressions entering in equation (19) are presented

$$\beta\int_0^x \frac{y^2\,F(y)}{(y+m^2)^2}\,dy \;=\; -x^2\,\frac{d^3}{dx^3}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+$$

$$+\,2x\,\frac{d^2}{dx^2}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;-\;2\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;12\,a_{1l}\,;$$

$$\beta\int_0^x \frac{y^3\,F(y)}{(y+m^2)^2}\,dy \;=\; -x^3\,\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;+\;3x^2\frac{d^2}{dx^2}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-$$

$$-\,6x\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;6\,x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\;-\;12\,a_{0l}\,;$$

$$\beta\int_0^x \frac{y^4\,F(y)}{(y+m^2)^2}\,dy \;=\; -x^4\,\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;+\;4x^3\frac{d^2}{dx^2}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-$$

$$-\,12x^2\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;24\,x^3\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\;-\;24\,x^2\,\frac{d^3}{dx^3}\Big(x^2\,F(x)\Big)\;+$$

$$+\,48\,x\,\frac{d^2}{dx^2}\Big(x^2\,F(x)\Big)\;-\;48\,\frac{d}{dx}\Big(x^2\,F(x)\Big)\;+\;48\,a_{-1}\,;$$

$$\beta\int_0^x\frac{y^5\,F(y)}{(y+m^2)^2}\,dy\;=\;-x^5\,\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;+\;5x^4\,\frac{d^2}{dx^2}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-$$

$$-\,20x^3\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;60\,x^4\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\;-\;120\,x^3\,\frac{d^3}{dx^3}\Big(x^2\,F(x)\Big)\;+$$

$$+\,360\,x^2\,\frac{d^2}{dx^2}\Big(x^2\,F(x)\Big)\;-\;720\,x\,\frac{d}{dx}\Big(x^2\,F(x)\Big)\;+\;720\,x^2\,F(x)\;-\;720\,a_{-2}\,;$$

$$\beta\int_x^\infty\frac{F(y)}{(y+m^2)^2}\,dy\;=\;\frac{d^3}{dx^3}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\,;\tag{57}$$

$$\beta\int_x^\infty\frac{y\,F(y)}{(y+m^2)^2}\;=\;x\,\frac{d^3}{dx^3}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;-\;\frac{d^2}{dx^2}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\,;$$

$$\beta\int_x^\infty\frac{y^2\,\log\,y\,F(y)}{(y+m^2)^2}\,dy\;=\;x^2\,\log\,x\,\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-\;(2x\log\,x+x)\,\times$$

$$\times\,\frac{d^2}{dx^2}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;(2\log\,x+3)\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;-$$

$$-\,2\,x\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\;+\;2\,\frac{d^3}{dx^3}\Big(x^2\,F(x)\Big)\,;$$

$$\beta\int_x^\infty\frac{y^2\,F(y)}{(y+m^2)^2}\,dy\;=\;x^2\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-\;2x\frac{d^2}{dx^2}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;+$$

$$+\,2\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\,;$$

$$\beta\int_x^\infty\frac{y^3\,\log\,y\,F(y)}{(y+m^2)^2}\,dy\;=\;x^3\,\log\,x\,\frac{d^3}{dx^3}\Big(x^2\frac{d^4}{dx^4}\Big(x^2F(x)\Big)\Big)\;-(3x^2\log\,x+x^2)\,\times$$

$$\times\,\frac{d^2}{dx^2}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;+\;(6\,x\log\,x+5x)\,\frac{d}{dx}\Big(x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\Big)\;-$$

$$-\,(6\log\,x+11)\,x^2\,\frac{d^4}{dx^4}\Big(x^2\,F(x)\Big)\;+\;6\,x\,\frac{d^3}{dx^3}\Big(x^2\,F(x)\Big)\;-\;6\,\frac{d^2}{dx^2}\Big(x^2\,F(x)\Big)\,.$$

For equation (38) one has to change sign afore β and change $F(x)$ for $\Psi(x)$.